Basic principles of steel structures

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Bending members (beam)

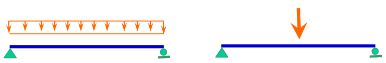
Outlines

- ✧ Introduction
- ✧ Resistance of cross-section of beams
- ✧ Lateral torsional buckling of beams
- ✧ Local buckling of plate element in beams
- ✧ Deflection of beams

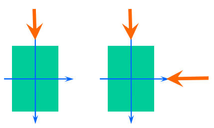
Bending members (beam)

introduction

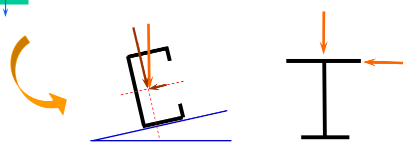
☑ state of loading



- bending moment
- shear force
- axial force
- combination of above



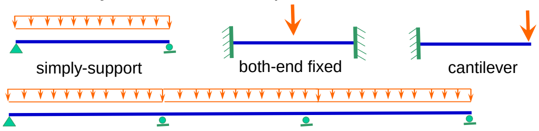
- bending about one axis
- bending about both axes (bi-axial bending)



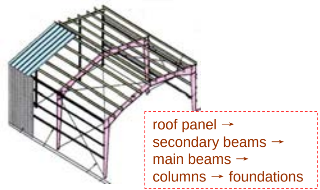
Bending members (beam)

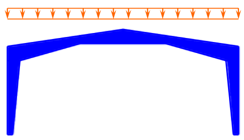
introduction

☑ boundary conditions and spans



☑ structural system and load transfer

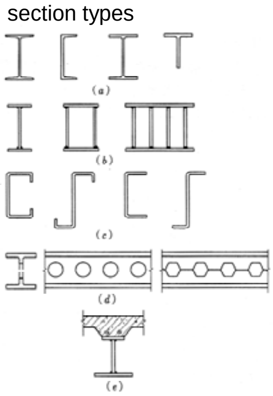




Bending members (beam)

introduction

☑ section types



solid-web section
(normal / light gauged)

↓

castellated beam

↓

composite beam

Bending members (beam)

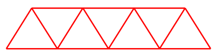
introduction

☑ section types

solid-web section

↓

open-web section

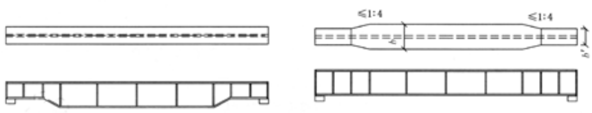


↓

laced built-up section
(truss)

↓

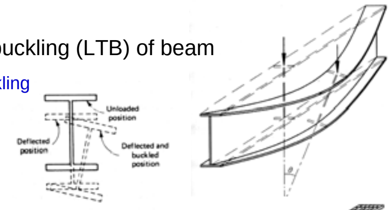
non-uniform section
(height, width, strength)



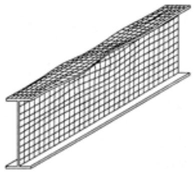
Bending members (beam)

introduction: failure modes

- ☑ strength failure (cross-section resistance)
  yield, fracture, fatigue
- ☑ Lateral-torsional-buckling (LTB) of beam
  flexural-torsional buckling



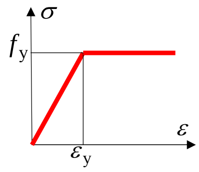
- ☑ Local buckling of plates in beam
  flange: compression
  web: compression, bending and shear
- ☑ Less rigidity of bending
  excessive deflection



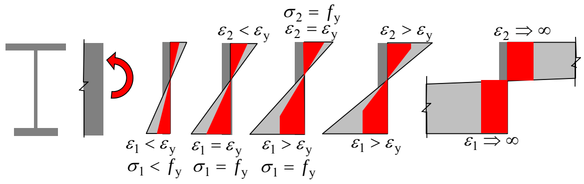
Cross-sectional resistance of beams

bending moment resistance (1)

- ☑ assumption
  - perfect elasto-plastic model
  - cross section remains plane during bending



- ☑ distribution of normal strain and stress



Cross-sectional resistance of beams

bending moment resistance (2)

☑ Criteria 1: yielding at extreme fibre

$$M_x \le M_{ex} \Rightarrow \sigma = \frac{M_x}{W_{xn}} \le f_d$$

elastic net (effective) section modulus

☑ Criteria 2: yielding on full section

$$M_x \le M_{px} \Rightarrow \frac{M_x}{W_{pxn}} \le f_d$$

plastic net section modulus

$$\gamma_{px} = \frac{M_{px}}{M_{ex}} = \frac{W_{px}}{W_x}$$

shape factor

☑ Criteria 3: yielding on partial section

$$M_x \le \gamma_x M_{ex} \Rightarrow \frac{M_x}{\gamma_x W_{xn}} \le f_d$$

$$1 < \gamma_x < \gamma_{px}$$

elastic net section modulus

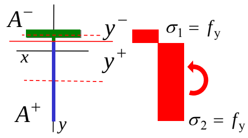
plastic adaptation factor

Memory: tension members with bending

concept of full plastic moment

☑ Assumption of stress distribution

- subjected to bending only, no tension
- stress at each point reach yield point
- yield point under tension and compression is the same



☑ Plastic neutral axis

$$N = 0 \Rightarrow A^+ f_y - A^- f_y = 0 \Rightarrow A^+ = A^-$$

The plastic neutral axis divides the cross section into two equal areas, and it may not coincide with the centroidal axis

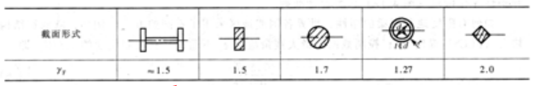
☑ (Full) plastic moment

$$M_{px} = A^+ f_y y^+ + A^- f_y y^- = (A^+ y^+ + A^- y^-) f_y$$

plastic modulus of a section: $W_{px} = A^+ y^+ + A^- y^-$ → $M_{px} = W_{px} f_y$

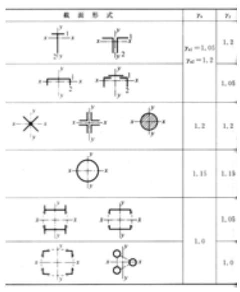
Review of section modulus

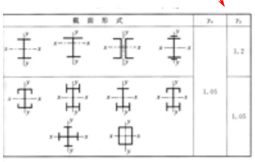
shape factor & plastic adaptation factor of cross-section



shape factor

plastic adaptation factor





Cross-sectional resistance of beams

bending moment resistance (3)

☑ Bi-axial bending: M_x, M_y

☑ Criteria 1: yielding at extreme fibre

$$\sigma = \frac{M_x}{W_{xn}} + \frac{M_y}{W_{yn}} \le f_d$$

☑ Criteria 2: yielding on full section

$$\frac{M_x}{W_{pxn}} + \frac{M_y}{W_{pyn}} \le f_d$$

☑ Criteria 3: yielding on partial section

$$\frac{M_x}{\gamma_x W_{xn}} + \frac{M_y}{\gamma_y W_{yn}} \le f_d$$

Cross-sectional resistance of beams
shear resistance

- ☑ shear stress:
  - mechanics of material: $\tau = \dfrac{V_y S_x}{I_x t}$
  - approximate value: $\tau = \dfrac{V_y}{A_w}$ (for I-shape or channel shape only)
- ☑ shear stress under bi-axial shear forces

$$\tau = \frac{V_y S_x}{I_x t} + \frac{V_x S_y}{I_y t}$$ ← algebraic or vector add?

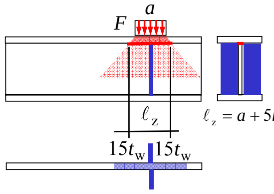
- ☑ shear resistance

$$\tau \le f_{vd}$$

Note: gross section or net section?

Cross-sectional resistance of beams
transverse force resistance

- ☑ local stress due to the transverse force



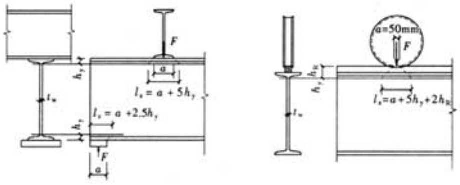
h_y :distance from load surface to the upper edge of the effective web width

$\ell_z = a + 5h_y + \{2h_R\}$ (height of rail)

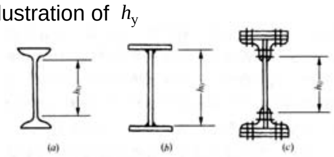
- ☑ Stress check $\sigma_c = \dfrac{F}{\ell_z \cdot t_w} \le f_d$
- ☑ Design of stiffeners
  - — place, thickness and width of stiffeners
  - — strength and stability of stiffeners and web around

Cross-sectional resistance of beams
transverse force resistance

- ☑ local stress due to the transverse force

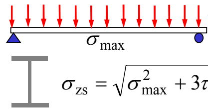


- ☑ illustration of h_y



Cross-sectional resistance of beams
state of stresses and equivalent stress

- ☑ state of combined stresses (SCS)
  - — stress state of existence of two or more stress components at same point / same load condition



Which section has SCS?

$\sigma_{zs} = \sqrt{\sigma_{max}^2 + 3\tau_{max}^2}$? *Which point has SCS?*

- ☑ Equivalent stress of beam $\sigma_{zs} = \sqrt{\sigma^2 + \sigma_c^2 - \sigma \cdot \sigma_c + 3\tau^2}$
  - — sign of stress component: tension positive and compression negative
- ☑ practical design equation

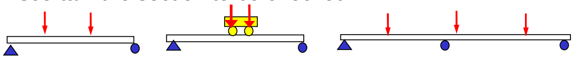
$\sigma_{zs} \le \beta_1 \cdot f_d$

$\beta_1 = 1$ Criteria of elasticity

$\beta_1 > 1$ Criteria of partially plastic adaptation

Cross-sectional resistance of beams
design procedure of beam strength

- ☑ Analytical model of the structure
  - Loading pattern and value, boundary conditions
- ☑ Internal force diagram under different load cases
  - Bending moment and shear forces
- ☑ Ascertain the section to be checked

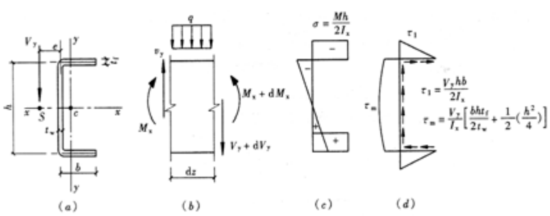


- ☑ Ascertain the computing point

- ☑ Calculate the sectional properties
- ☑ Calculate the nominal stress and equivalent stress
- ☑ Strength check

Lateral-torsional buckling of beams
preparation: shear centre

- ☑ concept of shear centre



$$M_z = \frac{V_y bh}{2I_x} \times \frac{bt_f}{2} \times h$$

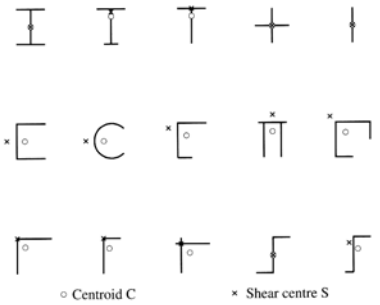
$$M_z = V_y e$$

⇒ $e = \dfrac{b^2h^2t_f}{4I_x}$

Lateral-torsional buckling of beams

preparation: shear centre

☑ centroid and shear centre

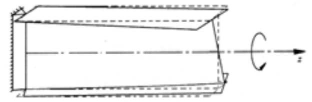


Lateral-torsional buckling of beams

preparation: free torsion

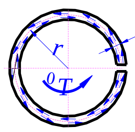
☑ free torsion

1. shear stress on section due to free torsion
2. uniform torsional angle along the member



$$M_k = GI_t\theta'$$

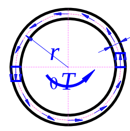
open-section



$$\tau = \frac{M_z t}{I_t}$$

$$I_t = \frac{1}{3}\sum_{i=1}^{n} b_i t_i^3$$

closed-section



$$\tau = \frac{M_z}{2A_0 t}$$

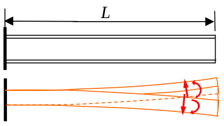
$$I_t = \frac{4A_0^2}{\oint \frac{ds}{t}}$$

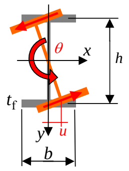
Lateral-torsional buckling of beams

preparation: restrained torsion

☑ restrained torsion and warping

Example:
cantilevered I-shape beam under end torsional moment



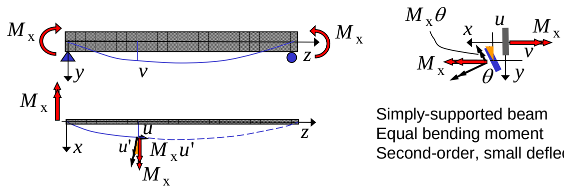


$$u = 0.5h\theta$$
$$\phi_y = u'' = 0.5h\theta''$$
$$M_y = -EI_y u'' = -0.5EI_y h\theta''$$
$$B_\omega = M_y h = -0.5EI_y h^2\theta''$$
$$I_\omega = 0.5I_y h^2 \approx b^3 t_f h^2 / 24$$
$$B_\omega = -EI_\omega\theta''$$
$$V_y = dM / dz = -0.5EI_y h\theta'''$$
$$M_\omega = V_y h = -0.5EI_y h^2\theta'''$$
$$M_\omega = -EI_\omega\theta''' \qquad M_k = GI_t\theta'$$
$$M_\omega + M_k = M_T$$

Lateral-torsional buckling of beams

differential equations for elastic LTB of beam

☑ equilibrium and deflection of LTB of beam



Simply-supported beam
Equal bending moment
Second-order, small deflection

☑ differential equations: flexural-torsional buckling

$$EI_x v'' + M_x = 0$$
$$EI_y u'' + M_x\theta = 0$$
$$EI_\omega\theta''' - GI_t\theta' + M_x u' = 0$$

$$EI_x v'' + Nv - Nx_0\theta = 0$$
$$EI_y u'' + Nu - Ny_0\theta = 0$$
$$EI_\omega\theta''' - GI_t\theta' - Nx_0 v' + Ny_0 u' + (Nr_0^2 - R)\theta' = 0$$

Lateral-torsional buckling of beams

solution of LTB for simply-supported beam with uniform bending

$$EI_y u'' + M_x\theta = 0 \qquad \textbf{(6-46b)}$$
$$EI_\omega\theta''' - GI_t\theta' + M_x u' = 0 \qquad \textbf{(6-46c)}$$

Substituting Equ.(-b) into Equ.(-c), we get differential equation about θ

$$EI_\omega\theta^{IV} - GI_t\theta'' - (M_x^2 / EI_y)\theta = 0 \qquad \textbf{(6-47)}$$

$$\theta = C\sin(n\pi z / \ell) \qquad \textbf{(a)}$$

boundary conditions: $\theta_0 = \theta_l = \theta_0'' = \theta_l'' = 0$

Substituting Equ.(a) into Equ.(6-47), we get

$$\left(\frac{EI_\omega n^4\pi^4}{\ell^4} + \frac{GI_t n^2\pi^2}{\ell^2} - \frac{M_x^2}{EI_y}\right)\cdot C\cdot\sin\frac{n\pi z}{\ell} = 0 \qquad \textbf{(b)}$$

Then we have

$$M_x \Rightarrow M_{crx} = \frac{\pi^2 EI_y}{\ell^2}\sqrt{\frac{I_\omega}{I_y}\left(1 + \frac{GI_t\ell^2}{\pi^2 EI_\omega}\right)} \qquad \textbf{(6-48)}$$

Lateral-torsional buckling of beams

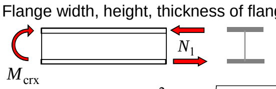
critical bending moment

☑ critical bending moment

$$M_{crx} = \frac{\pi^2 EI_y}{\ell^2}\sqrt{\frac{I_\omega}{I_y}\left(1 + \frac{GI_t\ell^2}{\pi^2 EI_\omega}\right)}$$

Assume web thickness is small but have enough stiffness to keep its shape, then we have the same eqution of critical bending moment

Flange width, height, thickness of flange and web for I-shape b, h, t_f, t_w



$$N_1 \approx \frac{M_{crx}}{h} = \frac{\pi^2 EI_y}{\ell^2}\sqrt{\frac{I_\omega}{I_y h^2}\left(1 + \frac{GI_t\ell^2}{\pi^2 EI_\omega}\right)}$$

$$\frac{I_\omega}{h^2} = \frac{b^3h^2t_f}{24\cdot h^2} = 0.25I_y, \qquad \frac{G}{\pi^2 E} \approx 0.039, \qquad \frac{I_t\ell^2}{I_\omega} \approx \frac{2bt_f^3}{3}\frac{24\ell^2}{b^3h^2t_f} = 16\frac{t_f^2\ell^2}{b^2h^2}$$

if $\frac{t_f^2\ell^2}{b^2h^2} \approx 1$ $\qquad N_1 \approx 0.64\frac{\pi^2 EI_y}{\ell^2}$? $> 0.5\frac{\pi^2 EI_y}{\ell^2}$

Lateral-torsional buckling of beams

critical bending moment with different boundary and loading

☑ Effect of boundary conditions

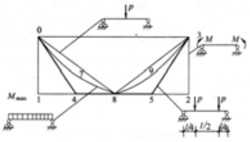
$$M_{crx} = \frac{\pi^2 EI_y}{(\mu_y \ell)^2}\sqrt{\frac{I_\omega}{I_y}\left[\frac{\mu_y^2}{\mu_\omega^2} + \frac{GI_t(\mu_y \ell)^2}{\pi^2 EI_\omega}\right]}$$

μ_y, μ_ω
Table 6-2 in pp.163

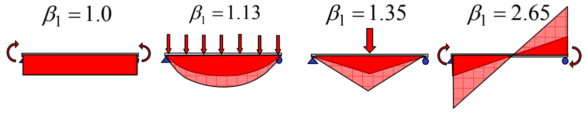
☑ Effect of loading pattern

$$M_{crx} = \beta_1 \cdot M_{ocrx}$$

M_{ocrx} : critical bending moment for beam under uniform bending



$\beta_1 = 1.0$ $\beta_1 = 1.13$ $\beta_1 = 1.35$ $\beta_1 = 2.65$



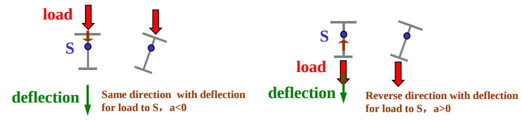
Lateral-torsional buckling of beams

critical bending moment with effects of section type and loading

☑ Effect of section types and loading point

$$M_{crx} = \beta_1 \frac{\pi^2 EI_y}{\ell^2}\left[\beta_2 a + \beta_3 B_y + \sqrt{(\beta_2 a + \beta_3 B_y)^2 + \frac{I_\omega}{I_y}\left(1 + \frac{GI_t \ell^2}{\pi^2 EI_\omega}\right)}\right]$$

a —distance from load point to shear centre



B_y —parameter indicating asymmetric degree for section

$$B_y = \frac{1}{2I_x}\int y(x^2 + y^2)dA - y_0$$

β_2— bending:0; UDL:0.46; CL at mid-span 0.55

β_3— bending:1; UDL:0.53; CL at mid-span 0.40

Lateral-torsional buckling of beams

parameters affect the LTB of beams

☑ Critical bending moment

$$M_{crx} = \frac{\pi^2 EI_y}{(\mu_y \ell)^2}\sqrt{\frac{I_\omega}{I_y}\left[\frac{\mu_y^2}{\mu_\omega^2} + \frac{GI_t(\mu_y \ell)^2}{\pi^2 EI_\omega}\right]}$$

$$M_{crx} = \beta_1 \frac{\pi^2 EI_y}{\ell^2}\left[\beta_2 a + \beta_3 B_y + \sqrt{(\beta_2 a + \beta_3 B_y)^2 + \frac{I_\omega}{I_y}\left(1 + \frac{GI_t \ell^2}{\pi^2 EI_\omega}\right)}\right]$$

—— rigidity of section
—— distance of lateral supports
—— section types (width of compressive flange)
—— loading pattern (bending moment)
—— loading point
—— boundary conditions

How about the initial imperfection?

Lateral-torsional buckling of beams

elasto-plastic lateral-torsional buckling of beams

☑ Elastic LTB

$$M_{crx} = \frac{\pi^2 EI_y}{\ell^2}\sqrt{\frac{I_\omega}{I_y}\left(1 + \frac{GI_t \ell^2}{\pi^2 EI_\omega}\right)}$$

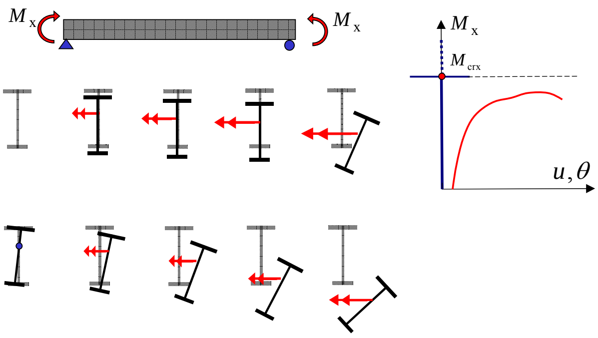
☑ Elasto-plastic LTB using tangent modulus theory

$$M_{crx} = \frac{\pi^2 (EI_y)_t}{\ell^2}\sqrt{\frac{(EI_\omega)_t}{(EI_y)_t}\left\{1 + \frac{[(GI_t)_t + \bar{K}]\ell^2}{\pi^2 (EI_\omega)_t}\right\}}$$

- short beam
- large residual stress

Lateral-torsional buckling of beams

ultimate capacity of beam with initial imperfections



Lateral-torsional buckling of beams

design equations

☑ Design equation for beams

$$M_x \le M_{crx} = \frac{M_{crx}}{M_{ex}} M_{ex} = \frac{\sigma_{crx}}{f_y} W_x f_y = \varphi_b W_x f_y$$

$$\Rightarrow \frac{M_x}{\varphi_b W_x} \le f_d$$

$$\varphi_b = \beta_b \cdot \frac{4320}{\lambda_y^2} \cdot \frac{Ah}{W_x}\left[\sqrt{1 + \left(\frac{\lambda_y t_1}{4.4h}\right)^2} + \eta_b\right] \cdot \frac{235}{f_y}$$

$$\frac{M_x}{\varphi_b W_x} + \frac{M_y}{W_y} \le f_d$$ ←bi-axial bending

φ_b : stability coefficient for beams
W_x: gross section modulus

☑ Conditions no need to check LTB of beams

1. A rigid decking is securely connected to compressive flange of beams
   How about the bottom flange under compression?
2. The max ratio of unsupported length to flange width is less than values listed in Table 6-3 in pp167

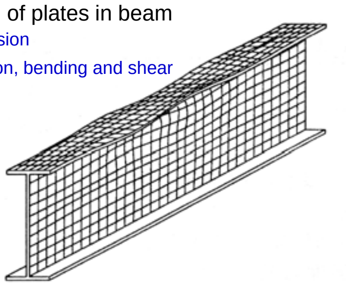
Local buckling of plates in beams

introduction

☑ Local buckling of plates in beam

flange: compression

web: compression, bending and shear



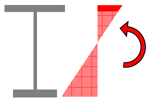
☑ Critical local buckling stress

$$\sigma_{cr} = \sqrt{\psi_t}\,\chi k \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2$$

Local buckling of plates in beams

critical local buckling stress of flange

☑ stress distribution of flange in beam



- shear stress is small
- normal stress is nearly uniform in flange

☑ critical local buckling stress

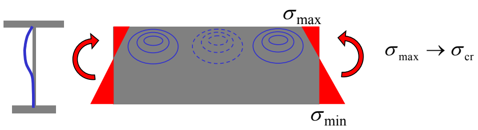
$$\sigma_{cr} = \chi \cdot k \frac{\pi^2 E}{12(1-\mu^2)} \cdot \frac{t^2}{b^2}$$

1. $\chi = 1$ ← flange having larger stiffness in beams
2. $k = 0.425$ ← outstand flange of I-section, *b* is flange outstand
   $k = 4.0$ ← flange supported along both edges of box-section
   b is the supported part in both edges

Local buckling of plates in beams

critical local buckling stress of web

☑ local buckling of plate under uneven compressive stress



☑ critical local buckling stress of plate simply supported at 4 edges

$$\sigma_{cr} = \chi \cdot k \frac{\pi^2 E}{12(1-\mu^2)} \cdot \frac{t^2}{b^2}$$

σ_{cr} : max. compressive stress (σ_{max}) while plate buckles

b, t: height and thickness of plate (web) h_w, t_w

k : stability factor of plate

$k = 4/(1-0.5\alpha)$ $\quad 0 \le \alpha \le 2/3$

$k = 4.1/(1-0.474\alpha)$ $\quad 2/3 < \alpha \le 1.4$

$k = 6\alpha^2$ $\quad 1.4 < \alpha \le 4$

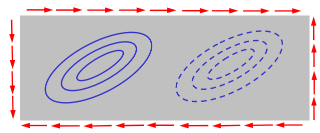
$\alpha = \dfrac{\sigma_{max} - \sigma_{min}}{\sigma_{max}}$

$\alpha = 2,\ k = 24, \chi = 1.61$

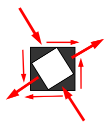
Local buckling of plates in beams

critical local buckling stress of web

☑ local buckling of plate in shear



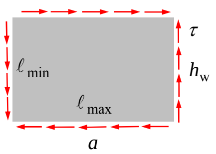
☑ why plate buckles in shear?



Local buckling of plates in beams

critical local buckling stress of web

☑ critical local buckling stress of plate in shear



$$\sigma_{cr} = k \frac{\pi^2 E}{12(1-\mu^2)} \cdot \frac{t^2}{b^2}$$

↓

$$\tau_{cr} = k \frac{\pi^2 E}{12(1-\mu^2)} \cdot \frac{t^2}{\ell_{min}^2}$$

$$k = 5.34 + \frac{4}{(\ell_{max}/\ell_{min})^2}$$

☑ critical local buckling stress of web of I-section in shear

$$\tau_{cr} = \chi \cdot k \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t_w}{h_w}\right)^2$$

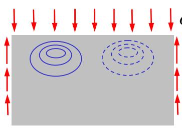
χ=1.24 $\quad k = 5.34 + 4(h_w/a)^2$ $\quad$ while $h_w/a \le 1$

$\quad k = 5.34(h_w/a)^2 + 4$ $\quad$ while $h_w/a > 1$

Local buckling of plates in beams

critical local buckling stress of web

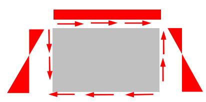
☑ local buckling and critical local buckling stress of plate under local compressive stress

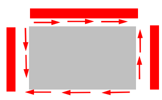


$\sigma_c \Rightarrow \sigma_{c,cr}$

$$\sigma_{c,cr} = C_1\left(100\frac{t}{h}\right)^2$$

☑ critical local buckling stress of plate under combined stress





$$\left(\frac{\sigma}{\sigma_{cr}} + \frac{\sigma_c}{\sigma_{c,cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 \le 1 \quad \textbf{Equ.(6-67)}$$

$$\left(\frac{\sigma}{\sigma_{cr}}\right)^2 + \frac{\sigma_c}{\sigma_{c,cr}} + \left(\frac{\tau}{\tau_{cr}}\right)^2 \le 1 \quad \textbf{Equ.(6-67a)}$$

$$\frac{\sigma}{\sigma_{cr}} + \frac{\sigma_c}{\sigma_{c,cr}} + \left(\frac{\tau}{\tau_{cr}}\right)^2 \le 1 \quad \textbf{Equ.(6-68)}$$

$$\frac{\sigma}{\sigma_{cr}} + \left(\frac{\sigma_c}{\sigma_{c,cr}}\right)^2 + \left(\frac{\tau}{\tau_{cr}}\right)^2 \le 1 \quad \textbf{Equ.(6-68a)}$$

Local buckling of plates in beams

design criteria of preventing local buckling of plates

Criteria 1: critical local buckling stress is larger than yield point

$$\sigma_{cr} > f_y$$

Criteria 2: critical local buckling stress is larger than critical overall buckling stress of member

$$\sigma_{cr} > \varphi_b \cdot f_y$$

Criteria 3: critical local buckling stress is larger than actual stress in plate

$$\sigma_{cr} > \sigma$$

Discussion: which criteria is the severest for local buckling prevention?

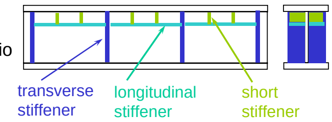
Local buckling of plates in beams

method to prevent plates from local buckling

☑ how to promote the critical local buckling stress?

$$\sigma_{cr} = k\frac{\pi^2 E}{12(1-\mu^2)} \cdot \frac{t^2}{b^2}$$

1. Modify boundary condition
2. Modify width-to-thickness ratio
   - increase thickness
   - setup stiffeners
   - which is better?

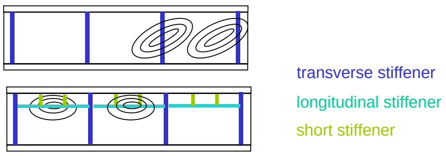


☑ decrease the actual stress in members (criteria 3)

Increase the height of section, thus makes the decrease of actual stress is faster than that of critical local buckling stress

Local buckling of plates in beams

stiffeners



Local buckling of plates in beams

design against local buckling of I-section

☑ outstand flange under compression

$$\sigma_{cr} = \frac{k\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{b}\right)^2 \geq 0.95 f_y$$

$$\frac{b}{t} \leq \sqrt{\frac{k\pi^2 E}{12(1-\upsilon^2)\times 0.95 f_y}} \leq \sqrt{\frac{0.425\times\pi^2\times 2.06\times 10^5}{12(1-0.3^2)\times 0.95 f_y}} = 18.8\sqrt{\frac{235}{f_y}}$$

consider the residual stress and initial imperfection,

$$\frac{b}{t} \leq \sqrt{\frac{k\pi^2\sqrt{\eta}E}{12(1-\upsilon^2)\times f_y}} \leq \sqrt{\frac{0.425\times\pi^2\times\sqrt{0.5}\times 2.06\times 10^5}{12(1-0.3^2)\times f_y}} = 15.4\sqrt{\frac{235}{f_y}}$$

consider the residual stress and initial imperfection, plus partial plasticity

$$\frac{b}{t} \leq \sqrt{\frac{k\pi^2\sqrt{\eta}E}{12(1-\upsilon^2)\times f_y}} \leq \sqrt{\frac{0.425\times\pi^2\times\sqrt{0.25}\times 2.06\times 10^5}{12(1-0.3^2)\times f_y}} = 13\sqrt{\frac{235}{f_y}}$$

Local buckling of plates in beams

design against local buckling of I-section

☑ web under bending, shear, local compression, respectively

1. subjected to bending moment

1.61, 170, restrained

1.23, 150, unrestrained

$$\sigma_{cr} = \frac{1.61\times 24\times\pi^2\times 2.06\times 10^5}{12(1-0.3^2)}\left(\frac{t_w}{h_w}\right)^2 \geq f_y \Rightarrow \frac{h_w}{t_w} \leq 174\sqrt{\frac{235}{f_y}}$$

Q/A: how about for the unsymmetric I-section with larger compressive flange?

2. subjected to pure shear   a / h=2.0

$$\tau_{cr} = \frac{1.24\times(5.34+1)\times\pi^2\times 2.06\times 10^5}{12(1-0.3^2)}\left(\frac{t_w}{h_w}\right)^2 \geq f_{vy} = 0.58 f_y \Rightarrow \frac{h_w}{t_w} \leq 104\sqrt{\frac{235}{f_y}}$$

3. subjected to local compression   a / h=2.0

$$\sigma_{c,cr} = 166\times\left(100\times\frac{t_w}{h_w}\right)^2 \geq f_y \Rightarrow \frac{h_w}{t_w} \leq 84\sqrt{\frac{235}{f_y}}$$

Local buckling of plates in beams

design against local buckling of I-section

☑ design procedure for real steelwork (no local buckling allowed)

1. For hot-rolled sections

   No need to check

2. For welded built-up sections

$$\frac{b_f}{t_f} \leq 15\sqrt{\frac{235}{f_y}} \quad \frac{h_w}{t_w} \leq 80\sqrt{\frac{235}{f_y}} \Rightarrow \text{O.K.}$$

$$\frac{b_f}{t_f} \geq 15\sqrt{\frac{235}{f_y}} \Rightarrow$$ modify flange section

$$\frac{h_w}{t_w} \geq 80\sqrt{\frac{235}{f_y}} \Rightarrow$$ modify web section, or setup stiffeners

3. subjected to local compression

$$80\sqrt{\frac{235}{f_y}} < \frac{h_w}{t_w} \leq 170(150)\sqrt{\frac{235}{f_y}} \Rightarrow$$ setup transverse stiffeners

$$\frac{h_w}{t_w} \geq 170(150)\sqrt{\frac{235}{f_y}} \Rightarrow$$ setup transverse, longitudinal stiffeners, plus short stiffeners if necessary

Local buckling of plates in beams
design against local buckling of I-section

☑ design procedure for real steelwork (no local buckling allowed)

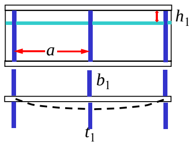
4. ascertain the space of stiffeners

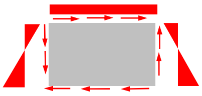
transverse stiffeners $0.5h_w \le a \le 2h_w$

longitudinal stiffeners $0.2h_w \le h_1 \le 0.25h_w$

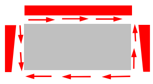
stability check for each grid

Method： Equ.(6-76)-(6.86)





$(\frac{\sigma}{\sigma_{cr}})^2 + (\frac{\tau}{\tau_{cr}})^2 + \frac{\sigma_c}{\sigma_{c,cr}} \le 1$



$\frac{\sigma}{\sigma_{cr}} + (\frac{\tau}{\tau_{cr}})^2 + (\frac{\sigma_c}{\sigma_{c,cr}})^2 \le 1$

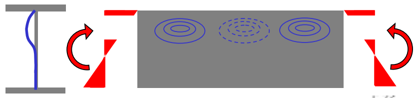
5. design of stiffeners

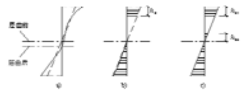
dimension, requirement of strength, rigidity and stability

Local buckling of plates in beams
design of bending members using post-buckling strength

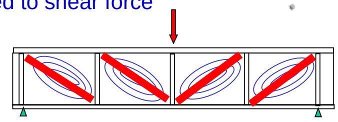
☑ Mechanism of post-buckling strength in bending

— subjected to bending moment





— subjected to shear force



Local buckling of plates in beams
design of bending members using post-buckling strength

☑ design principles

— elastic design allowing local buckling

— subjected to static load

☑ design method

— subjected to bending moment

concept of effective section

bending resistance on effective section, Equ.(6-87~90) pp177

— subjected to shear force

concept of tension field and truss-beam

discount the shear strength, Equ.(6-91~94) pp178-179

— subjected to combined bending and shear

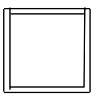
Equ.(6-95~100) pp179-180

Local buckling of plates in beams
application of post-buckling strength

☑ Portal frame

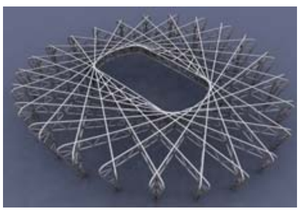
Local buckling of plates in beams
application of post-buckling strength

☑ National Stadium *bird nest*



☐ 1000×1000×20×20

☐ 600×600×10×10

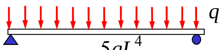


Local buckling of plates in beams
limit of width-to-thickness ratio and design method of beam

	width-to-thickness of flange	height-to-thickness of web
plastic design	9	70
partially plastic design	13	(150)
elastic design	15	(170)
design using post-buckling strength	(20)	250~300

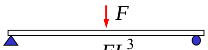
deflection of beams
calculation of elastic deflection and rigidity

- ☑ uniformly-distributed-load (UDL)



$$\delta = \frac{5qL^4}{384EI}$$

- ☑ concentrated load at mid-span (CL)



$$\delta = \frac{FL^3}{48EI}$$

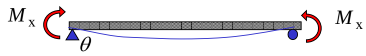
- ☑ multiple concentrated load (MCL)

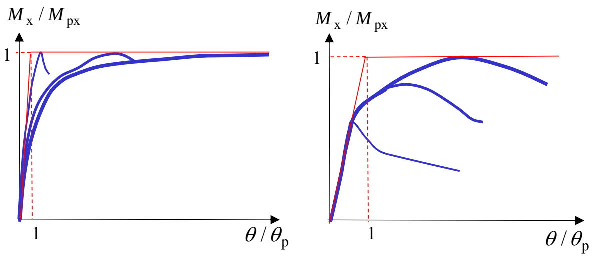
$$\delta \approx \frac{ML^2}{10EI}$$

- ☑ requirement of rigidity

$$\delta \le [\delta],\ [\delta] = L/1200 \sim L/150$$

deflection of beams
concept of deflection capacity





Design of bending members
summary

- ☑ Selection of section
- ☑ Calculation of section resistance (strength)
  - normal stress, shear stress, local compressive stress, combined stress
  - Using net section, except shear stress check
- ☑ Calculation of overall stability
  - Need to check the overall stability?
  - Ascertain the critical LTB bending moment
  - Using gross section
- ☑ Calculation of local buckling of plates
  - Calculation of width-to-thickness ratio, setup stiffeners, design of stiffeners
  - Calculation of beams using post-buckling strength
- ☑ Calculation of deflection