Basic principles of steel structures

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Members subjected to bending and axial compression (beam-column)

Outlines

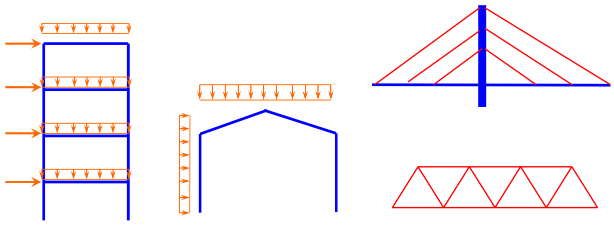
- ✧ Introduction
- ✧ Section capacity of beam-column
- ✧ Overall stability of beam-column
- ✧ Local buckling of plate element in beam-clomn
- ✧ Rigidity of compression members

Beam-column

introduction

☑ Beam-column members in structures

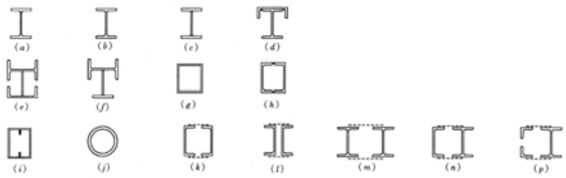
- column in frame structure
- rafter in portal frame
- longitudinal girder of cable-stayed bridge
- members in truss structures



Beam-column

introduction

☑ Structural shapes used in beam-column



☑ Selection of cross-section

Loading status	cross-section used
- major in compression, plus minor bending	*doubly symmetric section,* $\lambda x = \lambda y$
- major in bending about one axis	doubly / *singly symmetric section*
- bi-axial beam-column	

Beam-column

introduction: failure modes

☑ strength failure (section capacity)

yield, fracture, fatigue, connection

☑ stability failure (buckling resistance) of beam-column

Overall stability: in-plane instability and flexural-torsional buckling

Local buckling of plates in beam-column

Chord buckling in built-up beam-column

☑ Less rigidity of beam-column

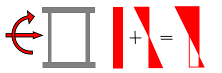
excessive deflection, loss of rigidity

Beam-column

section capacity of beam-column

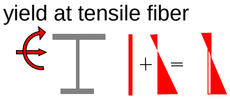
☑ Section capacity: normal stress dominated

— yield at compressive fiber



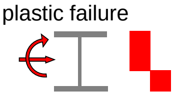
$$\frac{N}{A} + \frac{M_x}{W_{x1}} \le f_y$$

— yield at tensile fiber



$$\left| -\frac{N}{A} + \frac{M_x}{W_{x2}} \right| \le f_y$$

— plastic failure



$$A\left(\frac{N}{N_p} + n\right)^{\alpha} + B\left(\frac{M_x}{M_{px}} + m\right)^{\beta} = 1$$

Beam-column
section capacity of beam-column

practical design equations (net section)

☑ Criteria of elasticity

$$\left|\frac{N}{A_n} \pm \frac{M_x}{W_{xn}} \pm \frac{M_y}{W_{yn}}\right| \le f_d \qquad \frac{N}{A_n} \pm \frac{M_x}{W_{xn}} \pm \frac{M_y}{W_{yn}} \le f_d$$

☑ Criteria of partial plasticity

$$\left|\frac{N}{A_n} \pm \frac{M_x}{\gamma_x W_{xn}} \pm \frac{M_y}{\gamma_y W_{yn}}\right| \le f_d \qquad \frac{N}{A_n} \pm \frac{M_x}{\gamma_x W_{xn}} \pm \frac{M_y}{\gamma_y W_{yn}} \le f_d$$

☑ Criteria of full plasticity (bending about one axis)

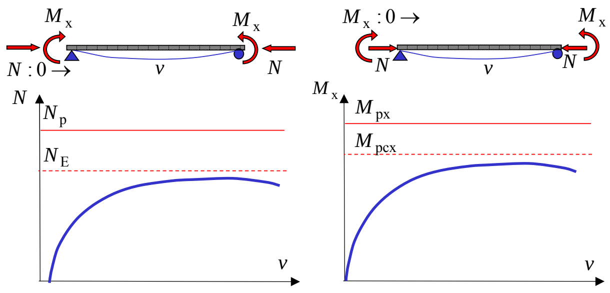
Equ.(4.18), pp85

if $\frac{N}{A_n f_d} \le 0.13 \qquad M_x \le W_{pnx} f_d$

if $\frac{N}{A_n f_d} > 0.13 \qquad M_x \le 1.15(1 - \frac{N}{A_n f_d})W_{pnx} f_d$

Beam-column
In-plane buckling of beam-column

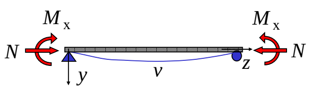
☑ In-plane overall stability



Why in-plane buckle s of beam-column?

Beam-column
In-plane buckling of beam-column

☑ differential equation of elastic buckling of beam-column



bending equilibrium about x-axis for all types section

$$EI_x v'' = -(M_x + Nv)$$

$$EI_x v'' + Nv + M_x = 0$$

differential equation while subjected to bending

$$EI_x v'' + M_x = 0$$

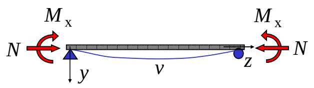
if $M_x = Ne_y$

For all types sections with illustrated supports, the differential equation will be

$$EI_x v'' + Nv = -Ne_y$$

Beam-column
In-plane buckling of beam-column

☑ solution for differential equation of elastic buckling of beam-column



$$EI_x v'' + Nv = -Ne_y$$

let $\alpha = \sqrt{N/EI_x}$ **we got,**

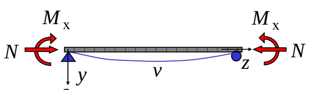
$$v = \frac{e_y}{\cos(\alpha\ell/2)}[\cos(\alpha\ell/2 - \alpha z) - \cos(\alpha\ell/2)]$$

$$v'' = -\frac{e_y\alpha^2}{\cos(\alpha\ell/2)}\cos(\alpha\ell/2 - \alpha z)$$

solution satisfy the boundary condition: $v_0 = v_\ell = 0 \qquad v_0'' = v_\ell'' = -\frac{M_x}{EI_x}$

Beam-column
In-plane buckling of beam-column

☑ curves for differential equation of elastic buckling of beam-column

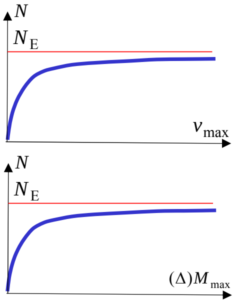


$$v = \frac{e_y}{\cos(\alpha\ell/2)}[\cos(\alpha\ell/2 - \alpha z) - \cos(\alpha\ell/2)]$$

$$v_{max} = e_y[\frac{1}{\cos(\alpha\ell/2)} - 1] \qquad v''_{max} = \frac{e_y\alpha^2}{\cos(\alpha\ell/2)}$$

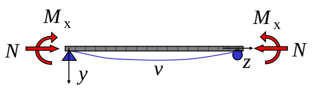
$$N \Rightarrow N_E = \frac{\pi^2 EI_x}{\ell^2}, \quad \cos\frac{\alpha\ell}{2} \Rightarrow \cos\frac{\pi}{2} = 0$$

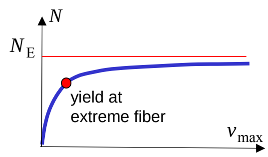
$$M_{max} = \frac{Ne_y}{\cos(\alpha\ell/2)} = \frac{M_x}{\cos(\alpha\ell/2)}$$



Beam-column
In-plane buckling of beam-column

☑ criteria of yielding at extreme fiber for beam-column





$$\frac{N}{A} + \frac{M_{max}}{W_x} = \frac{N}{A} + \frac{Ne_y}{W_x\cos(\alpha\ell/2)} \le f_y$$

$$\frac{N}{A}[1 + \frac{Ae_y}{W_x}\sec(\alpha\ell/2)] \le f_y$$

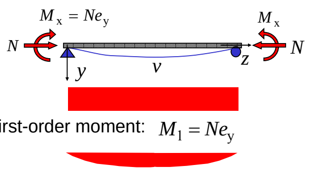
$$\sigma = \frac{N}{A} \le \frac{f_y}{1 + \varepsilon_{0y}\sec(\alpha\ell/2)}$$

where, $\varepsilon_{0y} = \frac{Ae_y}{W_x}$

Beam-column
In-plane buckling of beam-column

☑ second-order effects ($P-\Delta$ effects)

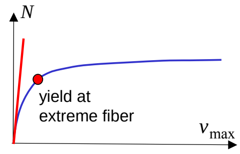
$$M_{\max}=\frac{Ne_y}{\cos(\alpha\ell/2)}=Ne_y\sec\frac{\alpha\ell}{2}$$



first-order moment: $M_1=Ne_y$

second-order moment: $M_2=Ne_y(\sec\frac{\alpha\ell}{2}-1)$

amplification factor considering second-order effect in elasticity: $\sec\frac{\alpha\ell}{2}$

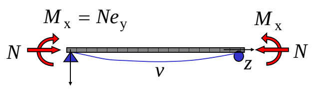


Before yield at extreme fiber, load-deflection curve is nonlinear due to second-order effect

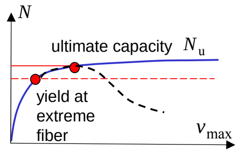
Criteria of yield at extreme fiber of in-plane buckling of beam-column is strength problem considering second-order effect, but it is not a section capacity problem due to its pertaining to the deflection of the whole member

Beam-column
In-plane buckling of beam-column

☑ ultimate capacity of beam-column



— develop plasticity after yielding at extreme fiber
— the increase of load effect (moment) is nonlinear
— the difference of increase between moment and section resistance leads to decrease of loads to maintain the bending equilibrium



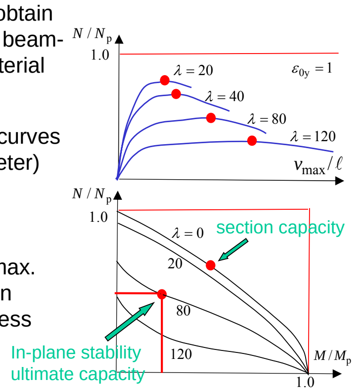
In-plane buckling of beam-column performs → extreme value of load-deflection curve → due to second-order effect of compression and deflection → In-plane buckling is not section capacity (strength) problem

Beam-column
In-plane buckling of beam-column

☑ numerical algorithm to obtain the ultimate capacity of beam-column considering material nonlinearity, pp191-193

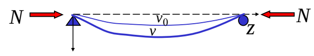
☑ axial force – deflection curves (slenderness as parameter) obtained by numerical algorithm

☑ curves of axial force – max. moment of beam-column while buckles (slenderness as parameter)



Beam-column
In-plane buckling of beam-column

☑ (beam-)column with initial imperfection



$$EI_x v''+Nv=Nv_0$$

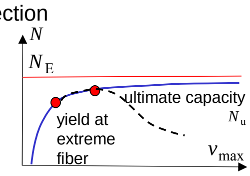
$$v_0=v_{0m}\sin(\pi z/\ell)$$

$$v=\frac{v_{0m}}{1-N/N_E}\sin(\frac{\pi z}{\ell})$$

$$M_{\max}=\frac{Nv_{0m}}{1-N/N_E}$$

amplification factor considering second-order effect in elasticity:

$$\frac{1}{1-N/N_E}$$



criteria of yield at extreme fiber:

$$\frac{N}{A}+\frac{Nv_{0m}}{W_x(1-N/N_E)}\le f_y$$

Beam-column
In-plane buckling of beam-column

☑ design formula for in-plane buckling resistance of solid-web beam-column

$$v_{\max}=e_y[\frac{1}{\cos(\alpha\ell/2)}-1]=\frac{M_x l^2}{8EI}\frac{8EI}{Nl^2}\left(\sec\frac{\alpha l}{2}-1\right)=\delta_0\left[\frac{2\left(\sec\frac{\alpha l}{2}-1\right)}{(\alpha l/2)^2}\right]$$

$$=\delta_0\times\left[1+1.028\frac{N}{N_{Ex}}+1.0316\left(\frac{N}{N_{Ex}}\right)^2+\dots\right]=\frac{\delta_0}{1-N/N_{Ex}}$$

$$\sec u=1+\frac{1}{2!}u^2+\frac{5}{4!}u^4+\frac{61}{6!}u^6+\dots$$

$$M_{\max}=Ne_y\sec(\alpha\ell/2)=Ne_y+Nv_{\max}=M_x+\frac{N\delta_0}{1-N/N_{Ex}}=\frac{M_x}{1-N/N_{Ex}}\left(1-N/N_{Ex}+\frac{N\delta_0}{M_x}\right)$$

$$=\frac{M_x}{1-N/N_{Ex}}\left[1+\left(\frac{N_{Ex}\delta_0}{M_x}-1\right)\frac{N}{N_{Ex}}\right]=\frac{\beta_{mx}M_x}{1-N/N_{Ex}}$$

Beam-column
In-plane buckling of beam-column

☑ design formula for in-plane buckling resistance of solid-web beam-column

$$\frac{N}{A}+\frac{M_{\max}}{W_{1x}}=\frac{N}{A}+\frac{\beta_{mx}M_x+Nv_0}{W_{1x}(1-N/N_{Ex})}=f_y$$

$M_x=0$

$$v_o=\frac{W_{x1}(Af_y-N_{ox})(N_{EX}-N_{ox})}{AN_{ox}N_{EX}}$$

$$\frac{N}{\varphi_x Af_y}+\frac{\beta_{mx}M_x}{W_{x1}f_y\left(1-\varphi_x\frac{N}{N_{EX}}\right)}=1$$

yield at extreme fiber

$$\frac{N}{\varphi_x A}+\frac{\beta_{mx}M_x}{\gamma_x W_{x1}\left(1-0.8\frac{N}{N'_{EX}}\right)}=f_y$$

partially plasticity

M_x — moment based on first-order analysis

0.8 — factor obtained from test and numerical results, considering the development of plasticity

Formula applied to "member"

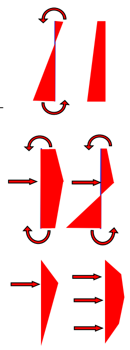
Beam-column

In-plane buckling of beam-column

☑ End moment factor (pp196)

Beam-column with sideway in the plane of bending / cantilever	Beam-column without sideway in the plane of bending
1.0	— no any transverse load $0.65+0.35\frac{M_2}{M_1}$
	— with end moment and transverse load: 1.0, 0.85
	— no end moment, but having transverse load: 1.0

$$\frac{N}{\varphi_x A}+\frac{\beta_{mx}M_x}{\gamma_x W_{x1}\left(1-0.8\frac{N}{N'_{EX}}\right)}=f_y$$

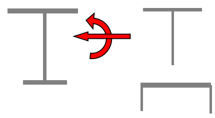


Beam-column

In-plane buckling of beam-column

☑ design formula for in-plane buckling resistance of unsymmetric solid-web beam-column with larger flange in compression

$$\left|-\frac{N}{A}+\frac{\beta_{mx}M_x}{\gamma_x W_{x2}\left(1-1.25\frac{N}{N'_{EX}}\right)}\right|\le f_y$$



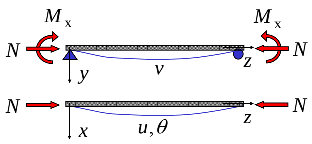
☑ design formula for in-plane buckling resistance of built-up section about virtual axis

$$\frac{N}{\varphi_x A}+\frac{\beta_{mx}M_x}{W_{x1}\left(1-\varphi_x\frac{N}{N'_{EX}}\right)}\le f_y$$

Beam-column

flexural-torsional buckling of beam-column

☑ Characteristic of flexural-torsional buckling of beam-column (out-of-plane buckling)



Similarity with overall buckling of beam：

occur flexural and torsion deformation out-of-plane

Difference with overall bucking of beam：

FTB occur under the combined axial force and moment

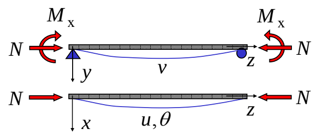
Difference with the FTB of axial compression member：

occur flexural and torsional deformation simultaneously for doubly symmetric section

Beam-column

flexural-torsional buckling of beam-column

☑ differential equation of elastic buckling of beam-column



doubly symmetric section
simply-support at ends
subjected to equal end moments

$$EI_y u'' + M_x\theta = 0$$

$$EI_y u^{IV} + Nu'' = 0$$

$$EI_y u^{IV} + Nu'' + M_x\theta'' = 0$$

$$EI_\omega\theta^{IV} - GI_t\theta'' + M_x u'' + (Nr_0^2 - \overline{R})\theta'' = 0$$

$$EI_\omega\theta^{IV} - GI_t\theta'' + (Nr_0^2 - \overline{R})\theta'' = 0$$

$$EI_\omega\theta''' - GI_t\theta' + M_x u' = 0$$

Beam-column

flexural-torsional buckling of beam-column

☑ solution for differential equation of elastic buckling of beam-column

$$EI_y u^{IV} + Nu'' + M_x\theta'' = 0$$

$$EI_\omega\theta^{IV} - GI_t\theta'' + M_x u'' + (Nr_0^2 - \overline{R})\theta'' = 0$$

Solution Procedure refers to pp197

Solution: $$\left(1-\frac{N}{N_{Ey}}\right)\left(1-\frac{N}{N_\theta}\right)-\frac{M_x^2}{M_{crx}^2}=0 \qquad \text{(7-18)}$$

where, $N_{Ey}=\pi^2 EI_y/\ell_{oy}^2$

$N_\theta=(\pi^2 EI_\omega/\ell_{o\theta}^2+GI_t+\overline{R})/r_0^2$

Axial force satisfying Equ.(7-18) is the critical FTB force

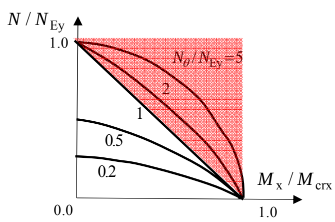
Discussion: beam-column buckles when compression reach N_{Ey}, N_θ?

Beam-column

flexural-torsional buckling of beam-column

☑ illustration of critical flexural-torsional-buckling resistance of beam-column

Solution: $$\left(1-\frac{N}{N_{Ey}}\right)\left(1-\frac{N}{N_\theta}\right)-\frac{M_x^2}{M_{crx}^2}=0$$



For most members in real structures:

$N_\theta/N_{Ey}>1$

That is ,

$$\frac{N}{N_{Ey}}+\frac{M_x}{M_{crx}}=1$$

It is the lower limit for flexural-torsional buckling of beam-column

Beam-column
flexural-torsional buckling of beam-column

☑ design formula for flexural-torsional buckling resistance of beam-column

Theoretical value (lower limit) for beam-column without imperfection: $\frac{N}{N_{Ey}}+\frac{M_x}{M_{crx}}=1$

Comparison between theoretical formula and real steelwork:
— not doubly-symmetric section
— elasto-plastic period
— coupling of differential equation due to initial imperfection

design formula:

$$\frac{N}{\varphi_y \cdot A}+\frac{\beta_{tx}\cdot M_x}{\varphi_b \cdot W_x}\le f_d$$

Beam-column
flexural-torsional buckling of beam-column

☑ design formula for flexural-torsional buckling resistance of beam-column

$$\frac{N}{\varphi_y \cdot A}+\frac{\beta_{tx}\cdot M_x}{\varphi_b \cdot W_x}\le f_d$$

— buckling out-of-plane (out-of-plane stability factor ϕ_y)
— moment used is the max. moment in segment to be checked
— end moment factor for FTB has the same meaning as that in-plane buckling pp.199
— stability factor of beam should be calculated as beam subjected to equal bending moment

☑ design formula for flexural-torsional buckling resistance of beam-column with closed section

$$\frac{N}{\varphi_y \cdot A}+\eta\frac{\beta_{tx}\cdot M_x}{\varphi_b \cdot W_x}\le f_d$$

η —1.0 for open section
0.7 for closed section

Beam-column
overall stability of built-up beam-column

☑ design formula of in-plane buckling resistance of beam-column about virtual axis

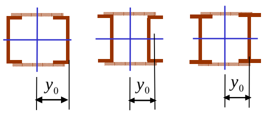
$$\frac{N}{\varphi_x A}+\frac{\beta_{mx}M_x}{W_{x1}\left(1-\varphi_x\frac{N}{N'_{EX}}\right)}\le f_y$$

comparison with solid-web beam-column

$$\frac{N}{\varphi_x A}+\frac{\beta_{mx}M_x}{\gamma_x W_{x1}\left(1-0.8\frac{N}{N'_{EX}}\right)}\le f_y$$

How to calculate the section modulus (compression)?

$$W_{x1}=I_x/y_0$$



Beam-column
overall stability of built-up beam-column

☑ design formula of out-of-plane buckling resistance of beam-column subjected to bending about virtual axis

No need if chord stability is ok

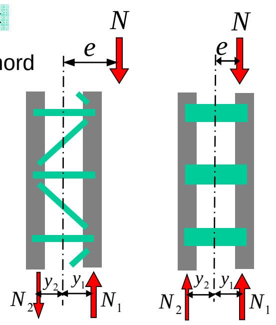
☑ axial force loaded on each chord

$e=\frac{M}{N}$

M is the max. moment in member

$$N_1=N\frac{y_2+e}{y_1+y_2}$$

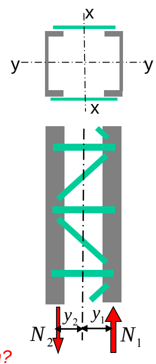
$$N_2=N-N_1$$



Beam-column
overall stability of built-up beam-column

☑ buckling resistance of chord in laced beam-column

1. Each chord performs as member pinned at two ends and withstands the force N_1 or N_2
2. Effective length:
   in-plane: length of adjacent lacing joints
   $\frac{N_1}{\varphi A_1}\le f_d$ $\lambda_1=\frac{l_1}{i_1}$
   out-of-plane: lateral supported length
3. Segments of member:
   in-plane: M along the member
   out-of-plane: max. M in each segment

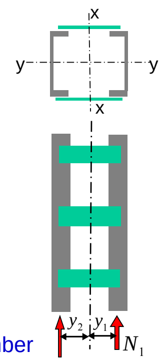


Q/A: shall we consider the shear force in such beam-column?

Beam-column
overall stability of built-up beam-column

☑ buckling resistance of chord in battened beam-column

1. Each chord withstands the local bending moment and shear force, in addition to axial force N_1 or N_2
2. Shear force:
   shear force in battened beam-column
   $V=\max\{\Delta M_x/\Delta H,\ (Af_d/85)\sqrt{f_y/235}\}$
   V acts to each chord, generating M
3. Effective length:
   in-plane (beam-column):
   length of adjacent battening
   out-of-plane (compression member):
   lateral supported length of the member



Beam-column

overall stability of built-up beam-column

☑ design formula of in-plane buckling resistance of beam-column subjected to bending about actual axis

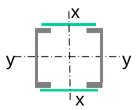
$$\frac{N}{\varphi_x A}+\frac{\beta_{mx}M_x}{\gamma_x W_x\left(1-0.8\,N/N'_{Ex}\right)}\le f$$

Same as that of solid-web beam-column

☑ design formula of out-of-plane buckling resistance of beam-column subjected to bending about actual axis

$$\frac{N}{\varphi_y A}+\eta\frac{\beta_{tx}M_x}{W_x}\le f$$

Same as that of solid-web beam-column, but….
- using equivalent slenderness while calculating φ_y
- φ_b equals to 1.0

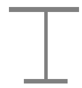


Beam-column

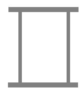
local buckling of plates in beam-column

☑ Flange subjected to compression
(limit of width-to-thickness ratio while no local buckling allowed)

outstand flange: $\frac{b}{t}\le 15\sqrt{\frac{235}{f_y}}$



edge-supported flange: $\frac{b}{t}\le 40\sqrt{\frac{235}{f_y}}$



Beam-column

local buckling of plates in beam-column

☑ Web
(limit of height-to-thickness ratio while no local buckling allowed)

I-shape section: $0\le\alpha_0\le 1.6,\quad \frac{h_w}{t_w}\le(16\alpha_0+0.5\lambda+25)\sqrt{\frac{235}{f_y}}$

$1.6<\alpha_0\le 2,\quad \frac{h_w}{t_w}\le(48\alpha_0+0.5\lambda-26.2)\sqrt{\frac{235}{f_y}}$

λ is in-plane slenderness, ranged 30~100

T-shape section:

web in tension: $\frac{h_w}{t_w}\le(15+0.2\lambda)\sqrt{\frac{235}{f_y}}$ **(hot rolled W split)**

$\frac{h_w}{t_w}\le(13+0.17\lambda)\sqrt{\frac{235}{f_y}}$ **(welded T section)**

web in compression: $\alpha_0\le 1.0,\quad \frac{h_w}{t_w}\le 15\sqrt{\frac{235}{f_y}}$

$\alpha_0>1.0,\quad \frac{h_w}{t_w}\le 18\sqrt{\frac{235}{f_y}}$

Beam-column

stiffness of beam-column

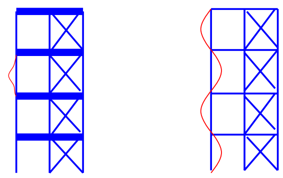
☑ slenderness

As compression member,
max. slenderness is 150~200

☑ In-plane deflection for beam-column

Lateral displacement

☑ Ascertain of slenderness in real steelwork



Design procedure of beam-column

summary

☑ Selection of member section

Section: requirement of overall stability, local buckling and ease to connect

☑ Strength: cross-section resistance

☑ Overall buckling resistance

Solid-web compression members: in-plane and out-of-plane
Laced or battened compression members: about virtual axis
chord stability: laced beam-column, battened beam-column
in-plane and out-of-plane

☑ local buckling resistance

Allowable ratio of width to thickness

☑ rigidity of beam-column