Basic principles of steel structures

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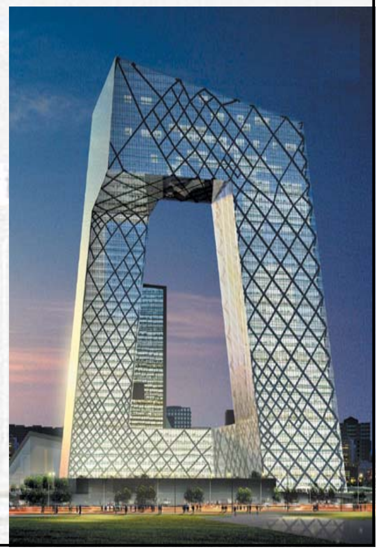
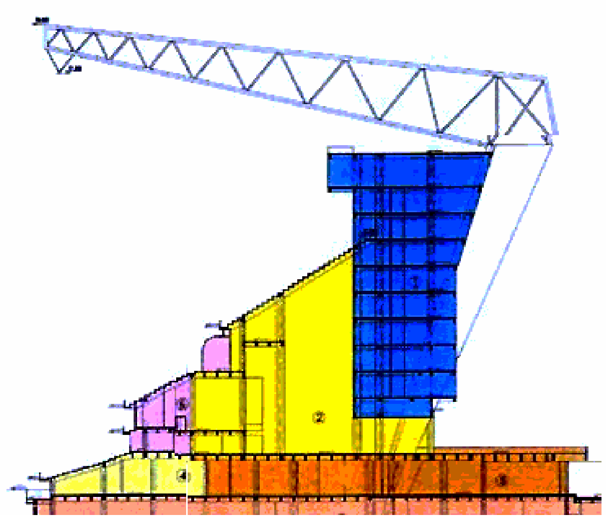


Tension members

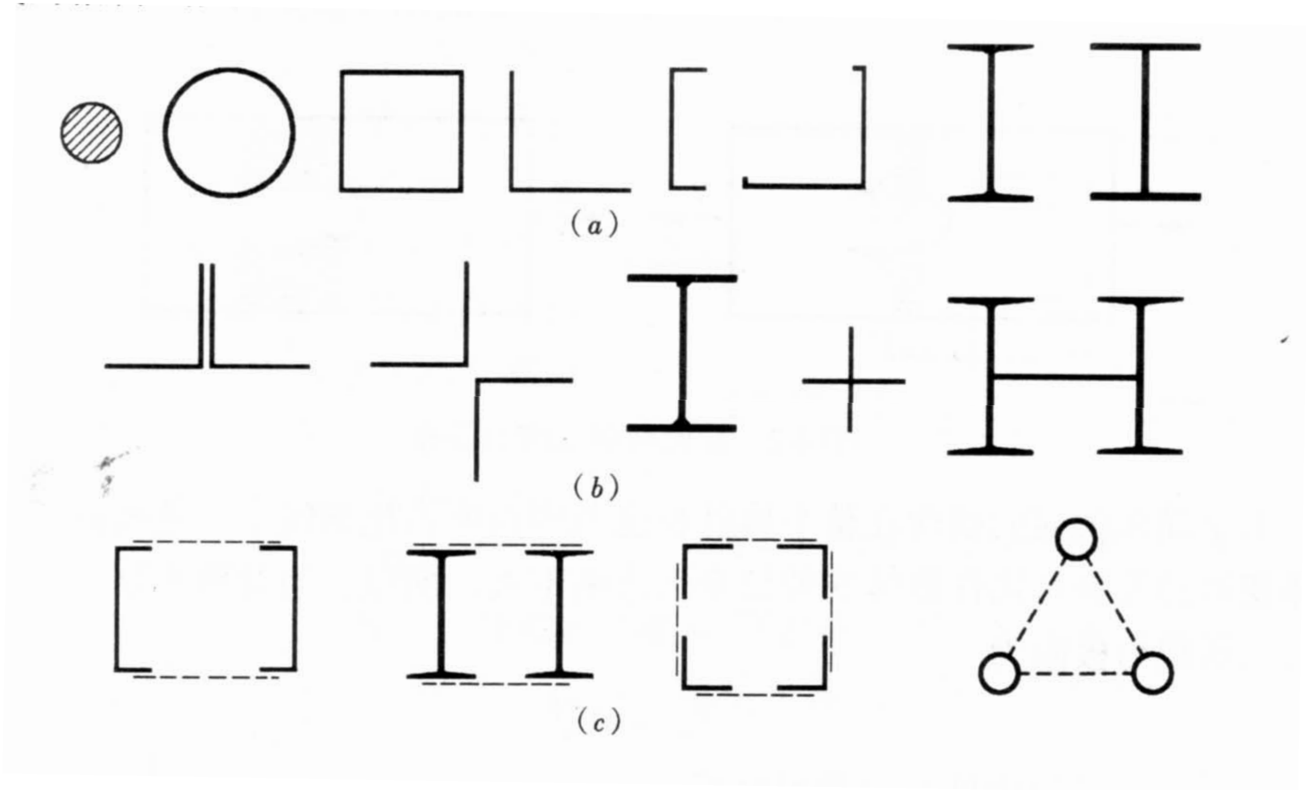
Outlines

- (axially loaded) tension member
- member subjected to tension and bending moment
- cable



Tension members

common used sections



Tension members

resistance of cross section

☑ Resistance of cross section for tension member

(1) for yielding in the gross section

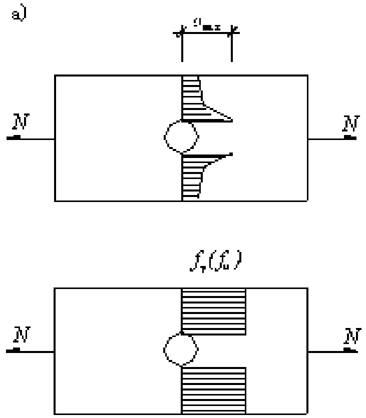
$$N_{\rm p} = A f_{\rm y}$$

(2) for fracture in the net section

$$N_{\rm u} = A_{\rm n} f_{\rm u}$$

Practical design equation

$$N \le \min(N_p, N_u)$$



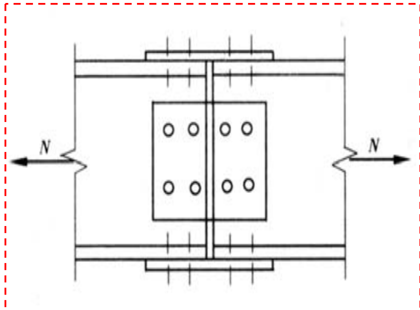
☑ Design equation in design code

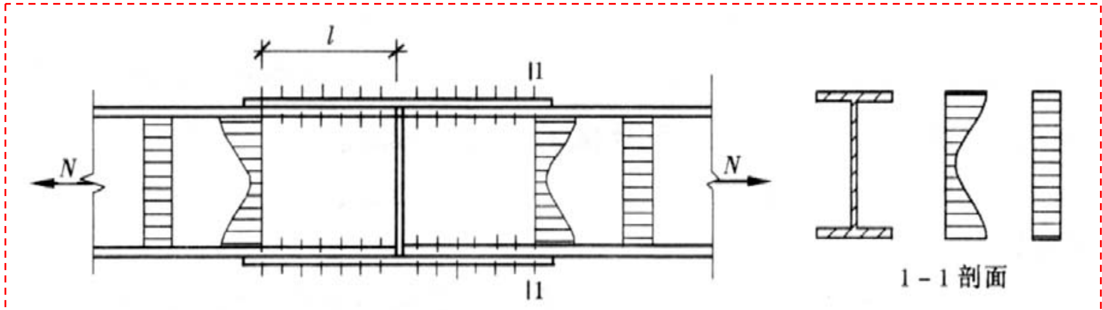
$$N \le A_{\rm n} f_{\rm d}, \quad f_{\rm d} = f_{\rm y} / \gamma_{\rm R} \Rightarrow \sigma = \frac{N}{A_{\rm n}} \le f_{\rm d}$$

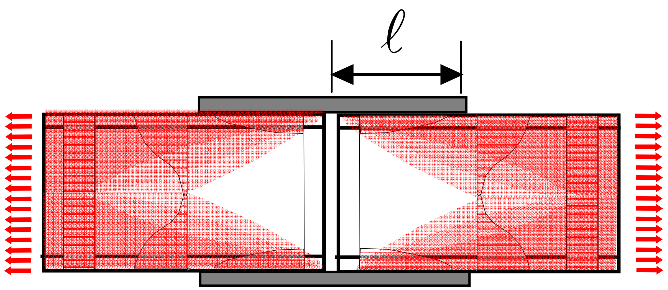
Tension members

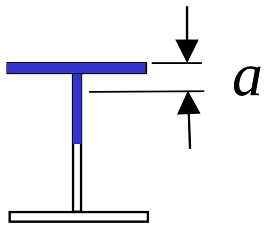
section efficiency

☑ Section efficiency: the ratio of the net section area enabling to transfer load to the area of net section



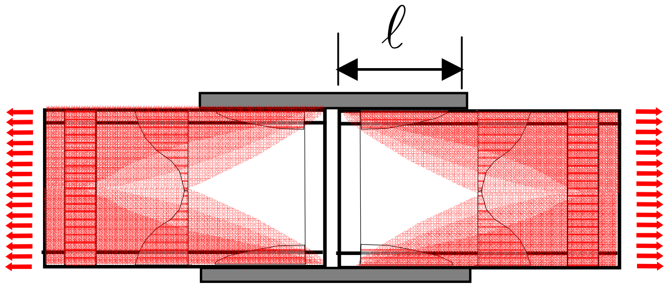


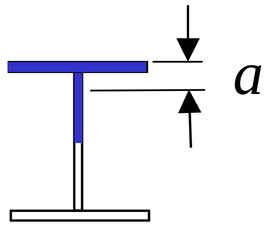




Tension members

section efficiency





☑ Factor of section efficiency

$$\eta = 1 - a / \ell$$

a —distance of centroid of connected area to the connection interface

ℓ —the length of connection

☑ Design equation

$$\frac{N}{\eta A_n} \leq f_d$$

Tension members

rigidity of members

☑ Rigidity of member

Ability to resist the deformation of member

Elastic rigidity used in real work

☑ Rigidity of tension member

Prevent excessive deformation during fabrication, transportation, erection and usage

In fact, it's bending deflection

☑ Check the rigidity of member: control the slenderness

$$\lambda \leq [\lambda]$$

$$\lambda = \max\{\lambda_x, \lambda_y\} = \max\{\mu_x \ell_x / i_x, \mu_y \ell_y / i_y\}$$

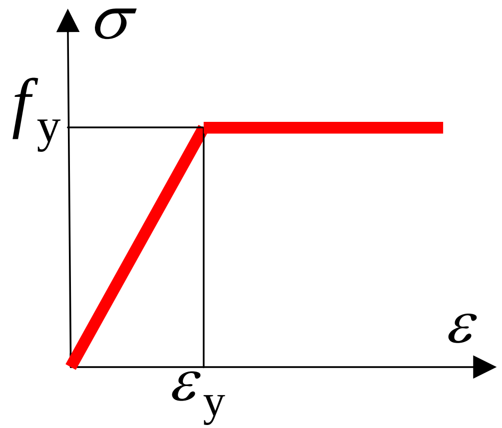
why not control EA/L for tension member?

Tension members with bending

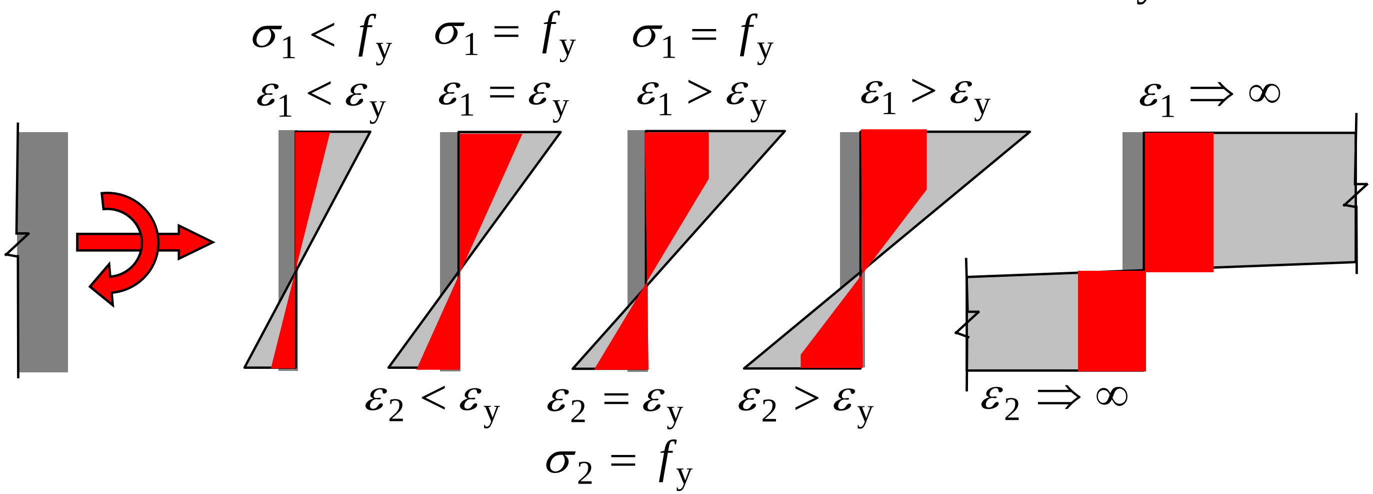
diagram of strain & stress distribution

☑ assumption

- perfect elasto-plastic model
- cross section remains plane during bending



☑ distribution of normal strain and stress



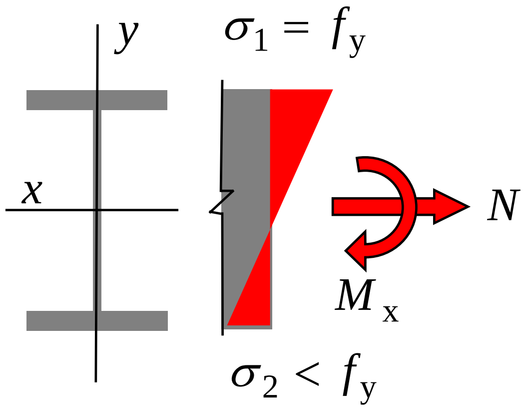
Tension members with bending

criteria of yielding at extreme fibre

☑ Description of failure criteria

The maximum stress at the top or bottom extreme fibre reach yield point (as limit state)

☑ Resistance of cross-section



$$\sigma_1 = \frac{N}{A} + \frac{M_x}{W_x} \le f_y$$

Let yield tension $N_p = A f_y$

yield moment $M_{ex} = W_x f_y$

then

$$\frac{N}{N_p} + \frac{M_x}{M_{ex}} \le 1$$

☑ Design equation

$$\frac{N}{A_n} + \frac{M_x}{W_{xn}} \le f_d$$

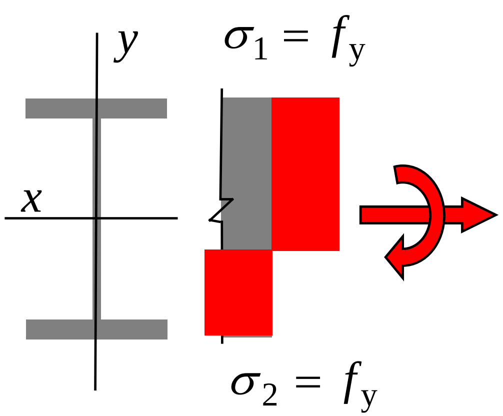
Tension members with bending

criteria of yielding on full section

☑ Description of failure criteria

The stress at each point of the section reach yield point

☑ Resistance of cross-section



Let yield tension $N_p = Af_y$

plastic moment $M_{px} = W_{px} f_y$

Then we get

$$A(\frac{N}{N_p})^{\alpha} + B(\frac{M_x}{M_{px}})^{\beta} = 1$$

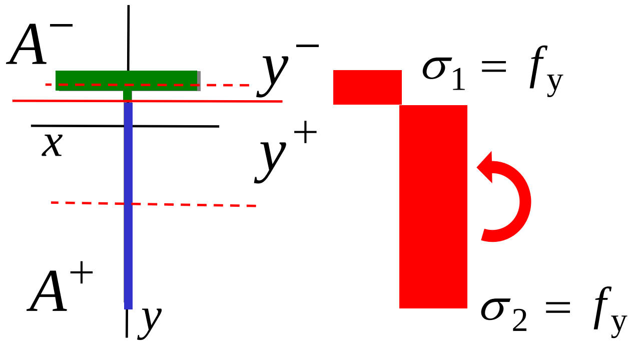
- coefficient A, B, α , β is related to section
- convex curve in N-M plane

Tension members with bending

concept of full plastic moment

☑ Assumption of stress distribution

- subjected to bending only, no tension
- stress at each point reach yield point
- yield point under tension and compression is the same



☑ Plastic neutral axis

$$N = 0 \Rightarrow A^+ f_y - A^- f_y = 0 \Rightarrow A^+ = A^-$$

The plastic neutral axis divides the cross section into two equal areas, and it may not coincide with the centroidal axis

☑ (Full) plastic moment

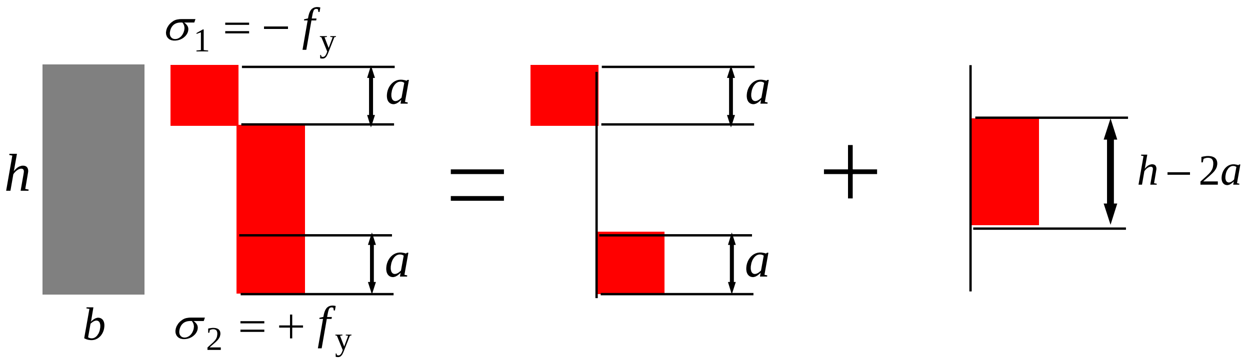
$$M_{px} = A^+ f_y y^+ + A^- f_y y^- = (A^+ y^+ + A^- y^-) f_y$$

section modulus of plastic bending: $W_{px} = A^+ y^+ + A^- y^-$

$$M_{px} = W_{px} f_y$$

Tension members with bending

interaction equation of tension and bending



equation of equilibrium:

$$N = (h-2a)bf_y = (1-2a/h)N_p \quad (1)$$

$$M_x = abf_y(h-a) = M_{px}4[a/h-(a/h)^2] \quad (2)$$

where, $N_p = bhf_y \quad M_{px} = (1/4)bh^2 f_y$

from equ. (1) $\quad a/h = 0.5(1-N/N_p)$

substitute to equ. (2) $\quad M_x/M_{px} = 4\times 0.5(1-N/N_p)[1-0.5(1-N/N_p)]$

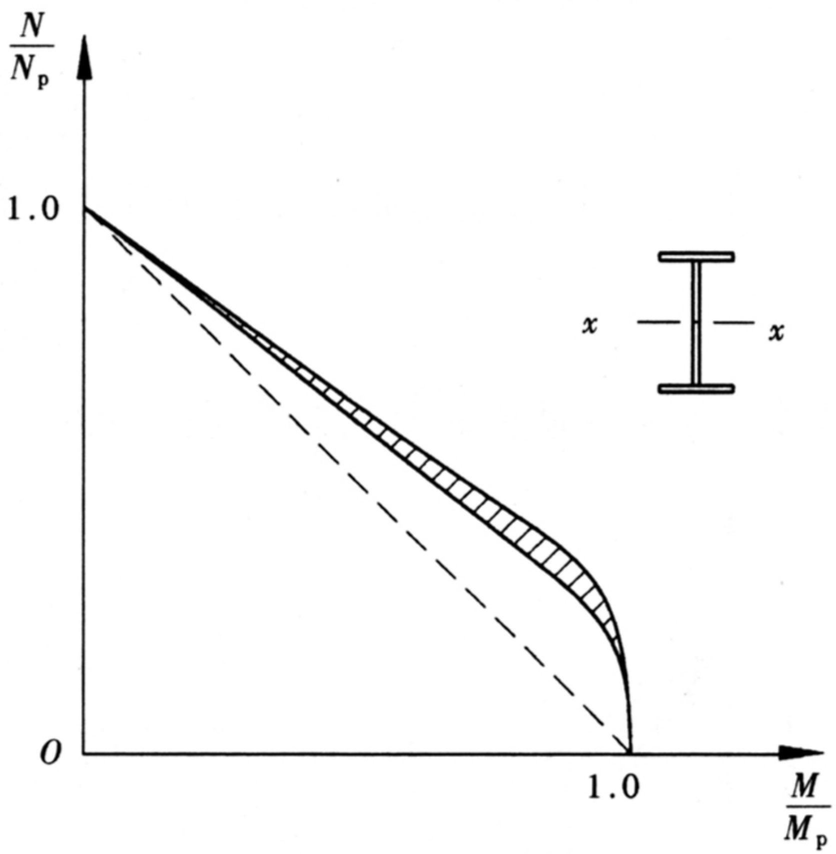
$$= 1-(N/N_p)^2$$

finally we get: $\quad (\frac{N}{N_p})^2 + \frac{M_x}{M_{px}} = 1$

Tension members with bending

criteria of yielding on full section

☑ Interaction of tension and bending for I shape



$$A\left(\frac{N}{N_p}\right)^{\alpha} + B\left(\frac{M_x}{M_{px}}\right)^{\beta} = 1$$

$$\frac{N}{N_p} > 0.13,\quad \frac{N}{N_p} + \frac{M_x}{1.15M_{px}} = 1$$

$$\frac{N}{N_p} \le 0.13,\quad \frac{M_x}{M_{px}} = 1$$

$$\Downarrow$$

$$\frac{N}{N_p} + \frac{M_x}{M_{px}} = 1$$

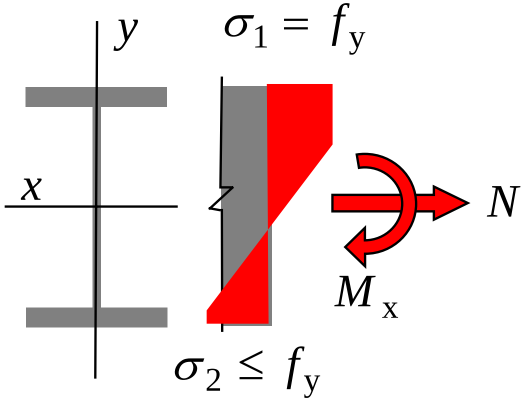
Tension members with bending

criteria of yielding on partial section

- ☑ Description of failure criteria

The stress at the top or/and bottom part reach yielding

- ☑ Resistance of cross-section



$$\frac{N}{A} + \frac{M_x}{\gamma_x W_x} \le f_y$$

⇓

$$\frac{N}{N_p} + \frac{M_x}{\gamma_x M_{ex}} \le 1$$

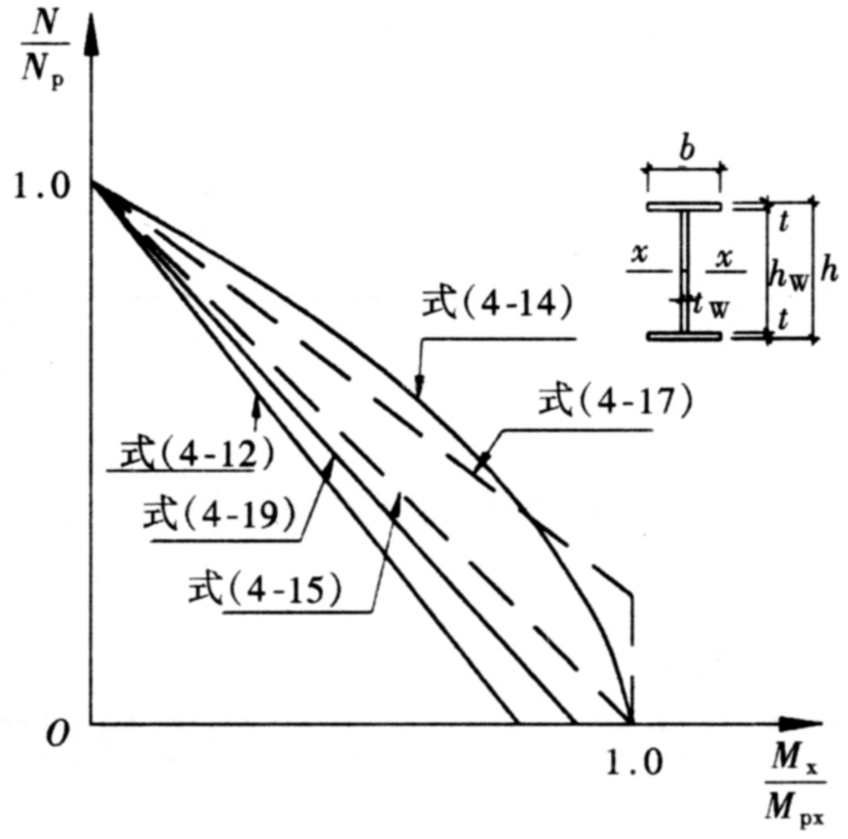
where, $\gamma_x \ge 1$

- ☑ Design equation

$$\frac{N}{A_n} + \frac{M_x}{\gamma_x W_{xn}} \le f_d$$

Tension members with bending

comparison of three criteria



Tension members with bending

rigidity and buckling

☑ Rigidity of members

- same as the tension member if tension is large
- check the deflection if bending is large

☑ Stability of members

If bending is large enough to make part area in compression

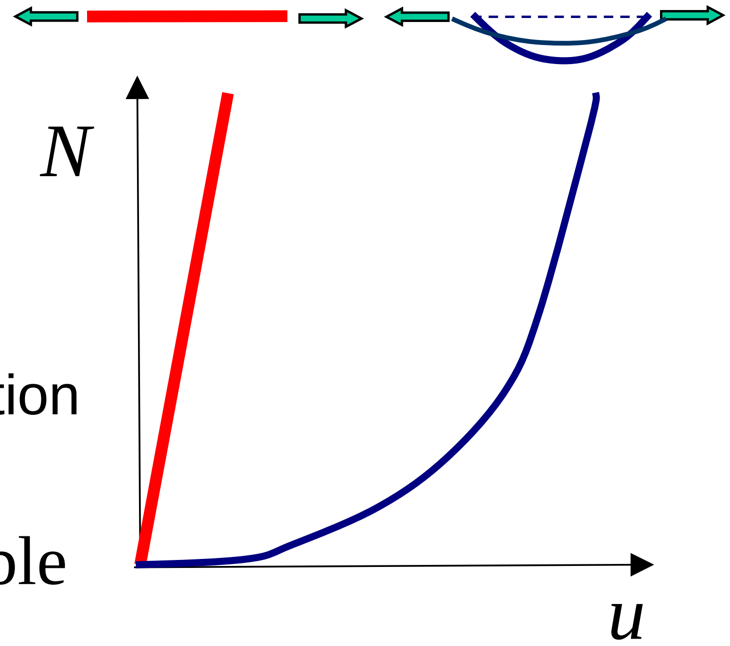
- possibility of overall buckling
- possibility of local buckling

Cable

axially loaded rigid bar and cable

☑ Relationship of force-deformation for rigid bar (in elasticity)

$$N = k \cdot u \quad \rightarrow k = \mathrm{Const}$$

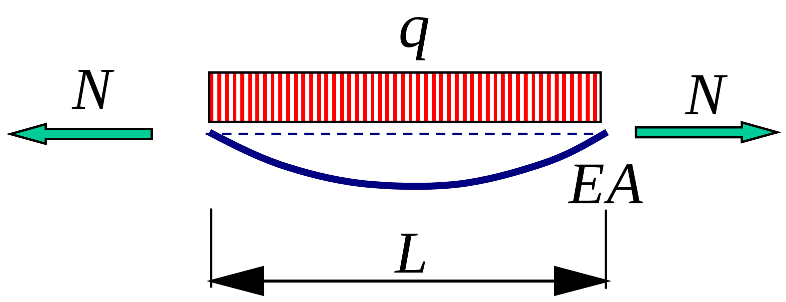


☑ Relationship of force-deformation for cable

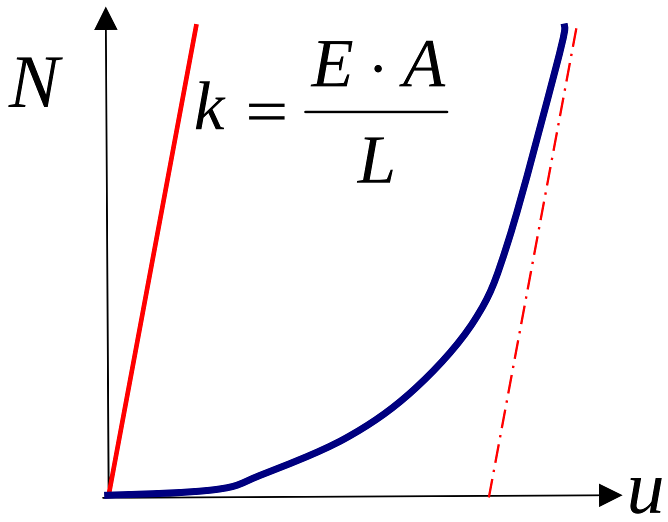
$$N = k(N) \cdot u \quad \rightarrow k = \mathrm{Variable}$$

Cable

stiffness equation



$$k = \frac{E \cdot A}{L} \cdot \frac{1}{1 + \dfrac{E \cdot A \cdot L^2 \cdot q^2}{12 N^3}}$$



Tension members

summary

☑ Resistance of cross-section

- net section (in the same section)
- section efficiency
- three failure criteria
- yield tension, yield moment and full plastic moment

☑ Rigidity of tension members

- slenderness in two principal axis

☑ Characteristic of cable